VI Iberoamerican Meeting on Geometry, Mechanics and Control

A SATELLITE OF

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INTERNATIONAL CONGRESS OF MATHEMATICIANS

August 13 - 17th, 2018
CIMAT, Guanajuato
Mexico

A conference in honor of James Montaldi

CIMAT
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Program at a glance

Program of the VI Iberoamerican Meeting on Geometry Mechanics and Control
13th-17th August 2018
(all talks will be held in CIMAT, Room D604)

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00-10:00	Registration		Rechtman		
		Dullin		Balseiro	Poutkaradze
10:00-11:00	Opening		Viviani		
	Chossat	Araathon	Coffee Break	Fassò	Vermeeren
11:00-12:00		Coffee Break	Koiller		Coffee Break
	Coffee Break	Montgomery		Coffee Break	Hernández
12:00-13:00	Schmah		Montaldifest	Gaset	
		Alishah	Excursion	Chacón	Shaddad
13:00-15:00	LUNCH	LUNCH		LUNCH	LUNCH
15:00-16:00	Zhilinskii	Ratiu		Cariñena	Martínez
16:00-17:00	García-Azpeitia	Fontaine		Vallejo	Jackman
	Coffee Break	Coffee Break		Poster Session	
17:00-18:00	Hatton	Ortiz			
18:00-19:00	Cabrera	Avendaño			
	Welcome Dinner				

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Tudor Ratiu, Shanghai Jiao Tong University, China
Miguel Rodríguez Olmos, UPC, Barcelona, Spain
Narciso Román-Roy, UPC, Barcelona, Spain
Cristina Stoica, Wilfrid Laurier University, Canada

Contents

Plenary and Contributed Talks

Perception of images by the visual cortex: geometry in neuroscience

Monday, 10:30

Pascal Chossat

Laboratoire J-A Dieudonné, Université Côte d'Azur - CNRS
Parc Valrose F - 06108 Nice, France

How does the brain process sensory information so as to restitute a coherent, global representation of the world (this is the Gestalt problem)?

Vision has been a much studied sensory system to investigate this problem: anatomical and functional experiments have provided a fairly detailed description of the visual system in the brain, at least from the retina to the primary visual cortex also named V1, which is the area in the visual cortex receiving the signal generated in the retina and processing first the "low-level" informations such as orientation of lines, contrast, spatial frequency etc. This information is however essentially of local nature. How does the visual cortex proceed to restitute from that a global geometrical information? This is a typical complex system with multiple spatial scales.

I shall first present a quick survey of the current state of knowledge about the anatomical and functional architecture of V1 and its mathematical idealization. Then I shall show how coupling this geometrical setup with physical modelling of neural networks can explain how visual global patterns are generated in the brain even in the absence of sensorial stimuli (visual hallucinations). If time permits I shall present some recent results about the mathematical treatment of this problem.

Geometric mechanics in medical imaging

Monday, 12:00

Tanya Schmah

University of Ottawa, Canada

Image "registration", or alignment, means finding corresponding points in multiple images. This is a problem that underlies many statistical and computational analyses of medical images. Given two images, a perfect correspondence would be a bijection, and it is usually reasonable to look for a diffeomorphism. How should we define, and find, the "best" one? This and related questions can be addressed by considering geodesics in diffeomorphism groups and their finite-dimensional subgroups. We will introduce this framework, discuss choices of metric and the role of the momentum vector field, and also mention recent interactions with the fields of optimal transport and deep learning.

Monday, 15:00

Topological transitions in isolated molecules: Energy band rearrangements

Boris Zhilinskii
Université du Littoral, Côte d'Opale, Dunkerque, France

During last 30 years a great progress has been achieved in theoretical description and experimental study of solid-state materials exhibiting different types of topological phase transitions. Investigation of topological phases of matter becomes now one of priority directions of modern science.

In my talk I'll show that isolated relatively small molecules demonstrate similar behavior. Dynamical regimes of isolated molecules can be characterized by topological invariants stable under small deformation of the Hamiltonian and showing topological phase transitions under variation of some control parameters. Energy bands and their rearrangements are manifestations of such topological phenomena. Concrete examples of molecular systems possessing energy bands characterized by first and second Chern classes will be discussed and generic rearrangements of energy bands under variation of control parameters are interpreted as topological phase transitions. Main contributions to the discussed subject are [1-6].

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Monday, 16:00

Nonlinear Vibrations in the Fullerene Molecule $C60$

Carlos García-Azpeitia
Facultad de Ciencias, UNAM, Mexico

The fullerene molecule is composed of 60 carbon atoms at the vertices of a truncated icosahedron. Kroto presents the definitive evidence of the existence of this molecule in 1986 and was awarded the Nobel price in chemistry for its discovery. The model of the fullerene molecule, consisting of 180 equations, is built on the framework of classical mechanics, where the force field comprise bond forces between carbon atoms connected by edges, and bond bending and torsion forces. The considered system of equations is equivariant under the action of the group $I \times O(3) \times O(2)$, where I denotes the full icosahedral group. We prove the existence of global families of periodic solutions (nonlinear vibrational modes of oscillation) from the equilibrium configuration. We use the gradient equivariant degree to obtain the topological classification of all spatio-temporal symmetries. We find that the solutions are standing and rotating waves that propagate along the molecule with icosahedral, tetrahedral, pentagonal and triangular symmetries. This is a joint work with Wieslaw Krawcewicz, Manuel Tejada-Wriedt, Haopin Wu.

Choice of gauge affects the curvature-holonomy relationship on non-abelian fiber bundles

Monday, 17:00

Ross L. Hatton
Oregon State University, USA

Given a principal connection on a trivial, principal fiber bundle, the net holonomy (fiber displacement) induced by a cyclic motion in the base space can be approximated by an integral of the curvature of the connection over a surface bounded by the cycle. This approximation is exact in the limit of infinitesimal cycle amplitudes (where it leads to Chow's theorem for controllability of nonholonomic systems) and when the fiber group is abelian (where it is well-known as Stokes' theorem).

On large-amplitude cycles on systems with non-abelian fiber groups, however, there can be significant error between the surface-integral-of-curvature and the actual fiber displacement. This error is associated with higher-order terms in the Magnus expansion (re-ordered Baker-Campbell-Hausdorff series for the path integral) which appear only in the presence of a nonabelian group.

We have found that the magnitude of this error is affected by gauge transformations applied to the system (for example, the choice of body frame on a multi-link system), and that the error can be minimized by working in a generalized Coulomb gauge (for a multi-link system, this places the body frame at a nonlinearly-weighted average of the link positions and orientations). Working in this gauge moves information from higher-order terms in the Magnus expansion to lower-order terms, and so makes the curvature-integral's truncation of the Magnus series more accurate of an approximation.

The transformation from an arbitrary gauge to the generalized Coulomb gauge can be found by applying a Hodge-Helmholtz decomposition to the system's connection, allowing for the physics to be worked out in a "convenient" gauge before transforming the problem into the error-minimizing gauge.

As a demonstration of this effect, we consider the motion of several undulatory locomoting systems. In the generalized Coulomb gauge, being able to apply the curvature-integral principle to large-amplitude motions allows us to identify gaits that maximize displacement per cycle with the zero-contours of the connection curvature on two-dimensional base spaces, and to extend this into a variational rule for maximal-displacement gaits over higher-dimensional base spaces. These results enable further analysis of gait optimality (taking into account dissipative and inertial costs of executing gait motions) and analysis of the motion of empirically-observed systems to which we have fit fiber-bundle/connection models.

Geometric splitting of variables in mechanical and control systems

Monday, 18:00

Alejandro Cabrera
Universidade Federal do Rio de Janeiro, Brazil

In this talk, we will discuss instances of mechanical and control systems in which the relevant variables can be split into 'horizontal' and 'vertical' components. We will show how this splitting makes curvature appear in the symplectic structure while simplifying the underlying hamiltonian. We will detail a class of examples based on the Pontryagin Maximum Principle applied to a control system with control in the acceleration (and, more generally, higher derivatives), as well as other examples.

Tuesday, 9:30

Monodromy in the Kepler Problem

Holger Dullin
University of Sidney, Australia
(joint work with Holger Waalkens, Groningen)

What could possibly be said about the Kepler Problem that is new? It is well known that this superintegrable system can be separated in different coordinate systems, and each such separation defines a distinct Liouville integrable system. We show that for separation in prolate spheroidal coordinates the resulting integrable system has Hamiltonian monodromy. This is a semi-toric system with two degrees of freedom on $S^2 \times S^2$ that is obtained by symplectic reduction of the S^1 action generated by the Kepler Hamiltonian. Analogous results are obtained for the corresponding quantum integrable system, where the eigenfunctions are spheroidal harmonics. Similar analysis can be done for many prominent superintegrable systems, for example the harmonic oscillator, the free particle or the geodesic flow on the sphere. In all these cases the resulting reduced systems correspond to well known special functions, but the quantum monodromy in their joint spectrum is reported in Phys. Rev. Lett. 120, 020507 (2018) for the first time.

Tuesday, 10:30

Singular reduction of the 2-body problem on the 3-sphere and the 4-dimensional Lagrange top

Philip Arathoon
University of Manchester, UK

For the motion of two attracting bodies in Euclidean space, the translational symmetry reduces the system to that of the usual Kepler problem. For two bodies on a sphere this is no longer the case as no such group of translational symmetries exist except for when the sphere in question is the 3-sphere. In this case, both left and right translations act as symmetries, and together generate the action of $SO(4)$. Furthermore, owing to the double cover of $SO(4)$, the dynamics for the 2-body problem on the sphere double cover the dynamics of a symmetric heavy top in 4 dimensions. The top is a generalised Lagrange top possessing an axis of symmetry within the body. The left and right $SO(3)$ reductions, about the line of gravity and the line of symmetry in the top, correspond under the double cover to the left and right translations of the 2-body problem. We investigate the geometry of the reduced manifolds and study their relative equilibria.

Tuesday, 11:30

The Kepler Cone and Hamilton-Jacobi theory

Richard Montgomery
UC Santa Cruz, USA

At zero energy, the Jacobi-Maupertuis metric yields a surface isometric to the cone over a circle of radius one-half, the cone point corresponding to collision with the sun. This perspective allows us to explicitly solve for the Buseman solutions of the Hamilton-Jacobi equation -solutions emphasized by Percino and Sanchez recently – and appear to have applicability to understanding weak KAM for the N-body problem near collisions.

Asymptotic Poincaré Maps along the edges of Polytopes and their Hamiltonian character

Tuesday, 12:30

Hassan Najafi Alishah
Universidade Federal de Minas Gerais, Brazil
(joint with Pedro Duarte and Telmo Peixe, Lisboa)

For a general class of flows on polytopes, we describe a method to encapsulate the asymptotic dynamics of the flow along the heteroclinic network formed by the polytope's edges. Using this method, we give sufficient conditions for the existence of normally hyperbolic stable and unstable manifolds associated to heteroclinic cycles along the polytope's edges. The main application of our results and in fact our main motivation is to study polymatrix replicator systems which contains several important models in Evolutionary Game Theory. For these systems, we also establish the Hamiltonian character, in the context of poisson geometry, of the asymptotic dynamics for Hamiltonian polymatrix replicators. We illustrate our results proving the existence of chaotic behavior in a Hamiltonian replicator system.

Group valued momentum maps and applications

Tuesday, 15:00

Tudor Ratiu
Shanghai Jiao Tong University, China

Persistence of stationary motion under explicit symmetry breaking perturbation

Tuesday, 16:00

Marine Fontaine
Universität Augsburg, Germany

Explicit symmetry breaking occurs when a dynamical system having a certain group of symmetries is perturbed in a way that the perturbation conserves only some symmetries of the original system. Simple examples include perturbing the rotational motion of small molecules by adding a weak magnetic field or perturbing a symmetric heavy top to one that is not symmetric. We give a geometric approach to study this phenomenon in the setting of equivariant Hamiltonian systems. A lower bound for the number of orbits of equilibria and orbits of relative equilibria which persist under a small perturbation is given. This work is in collaboration with James Montaldi.

Local aspects of foliations induced by vector fields with degenerate singularity

Tuesday, 17:00

Laura Ortiz
Instituto de Matemáticas, UNAM, Mexico

We will give an overview on the analytical and differential classification of foliations induced by vector fields with degenerated isolated singularity. We will start with a quick review of the classic results and then move towards degenerate cases. These will be reviewed in the analytical and differentiable cases in dimension two and, if time allows it, we will finish by stating some results for non-isolated singularities in a dimension greater than two.

Tuesday,
18:00

Geometric aspects of the theory of adiabatic Hamiltonian systems

Misael Avendaño Camacho
Universidad de Sonora, Mexico

We discuss some geometric aspects of the adiabatic theory of slow-fast Hamiltonian systems on general phase spaces with symmetry. The main observation is that an adiabatic Hamiltonian system can be regarded as a perturbed system on a Poisson fiber bundle associated with a horizontal deformation (a 2-cocycle) of the vertical Poisson structure. This gives rise to a link between the deformation theory for Poisson structure and the adiabatic perturbation theory on general fibered manifolds. Under natural symmetry assumptions, we present some normalization results for slow-fast Hamiltonian systems which are based on the averaging technique for Poisson connections and the following fact: averaging procedure with respect to a compact Lie group action on Poisson fiber bundle preserves the cohomology classes of horizontal 2-cocycles in the vertical Poisson-Lichnerowicz complex.

Wednesday,
9:00

The minimal set of a Kuperberg flow

Ana Rechtman
Instituto de Matemáticas, UNAM, Mexico

The Seifert Conjecture stated that every flow without stationary points on the three sphere has a periodic orbit. After many partial results, in 1994 K. Kuperberg proved that the Seifert Conjecture is false by inventing a construction of an aperiodic plug which is renowned for its simplicity, beauty and subtlety. A plug is a compact 3-manifold with boundary equipped with a flow satisfying additional conditions. The flow in a plug is assumed to be parallel to the "vertical" part of the boundary, so that it may be inserted in any coordinate chart of a 3-manifold M to modify the given flow on M, and changes only those orbits entering and leaving the "horizontal" faces of the plug. Another assumption on the flow in a plug is that there are orbits, which are said to be "trapped", which enter the plug and never exit. The closure of such an orbit limits to a compact invariant set contained entirely within the interior of the plug, thus the plug must contain at least one minimal set. I will present part of the study of the minimal set inside the Kuperberg plug. To my knowledge, this is the first explicit description of an exceptional minimal set of topological dimension 2 of flow. This is work in collaboration with Steve Hurder.

Lie-Poisson simulation of the dynamics of point vortices on a (rotating) sphere coupled with a background field

Wednesday, 10:00

Milo Viviani

Chalmers University of Technology, Sweden

One of the major unsolved problem of the Euler equations on a sphere is the long-time behaviour of an inviscid fluid when a certain initial vorticity is given [4],[6]. The biggest challenge in understanding and simulating this system is due to the presence of an infinite number of first integrals (Casimirs), which however don't provide complete integrability [7]. Moreover, the Hamiltonian nature of the equations suggests that much geometry is involved in them. They are in fact a Lie-Poisson system on $sdiff^*(\mathbb{S}^2)$ (the dual of the Lie algebra of divergence free vector fields) [1]. Special solutions to the Euler equations come from the fact that on $sdiff^*(\mathbb{S}^2)$ there exist non trivial finite dimensional coadjoint orbits, usually called point vortices [5]. However these orbits have physical interest only when the sphere is not rotating. When this is not true, the Euler equations become a coupled system of equations of a singular field of point vortices and a smooth continuous background vorticity [3].

Starting from [2],[8], we present an approximation of the coupled model based on the quantization of Kähler manifolds, which keeps the Hamiltonian Lie-Poisson structure of the equations. Moreover, with the techniques of the geometric integration, we provide a Lie-Poisson numerical scheme to solve the quantized model, preserving up to roundoff precision the discrete Casimirs and, up to the order of the method, the Hamiltonian. The conservation of the quantized first integrals provides a deep insight in the nature of the Euler equations and a better qualitative simulation of them.

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8. V. Zeitlin, Self-Consistent-Mode Approximation for the Hydrodynamics of an Incompressible Fluid on Non rotating and Rotating Spheres, Physical review letters, PRL 93 (2004)

Vortices on the Triaxial Ellipsoid

Special Lecture in honor of James Montaldi

Wednesday, 11:00

Jair Koiller

Departamento de Matematica, Universidade Federal de Juiz de Fora, Brazil

Point vortex dynamics figures among the many interests of James Montaldi. James and coworkers have worked on the sphere and surfaces of revolution since the early 2000's. Here we consider the problem of point vortices moving on the surface of a triaxial ellipsoid. We obtain the equations of motion by constructing a conformal map from the ellipsoid into the sphere. We focus on the case of a pair of opposite vortices. We present some results, numerical simulations, and some speculations, ranging from reasonable to wild. Joint work with Adriano Regis and Cesar Castilho.

Thursday, 9:30

On first integrals of nonholonomic systems

Paula Balseiro
Universidade Federal Fluminense, Brazil

In this talk we will discuss geometric aspects of nonholonomic systems with symmetries. In particular, we will study nonholonomic systems with "vertical symmetries" and see how the existence of these symmetries induce first integrals. This is a joint work with N. Sansonetto.

Thursday, 10:30

Moving energies and integrability of symmetric mechanical nonholonomic systems with affine constraints

Francesco Fassò
University of Padova, Italy

The appropriate 'energetic' integral for mechanical nonholonomic systems with linear non-homogeneous constraints appears to be not the energy, but a family of modifications of it called 'moving energies'. The conservation of a moving energy is related to the existence of symmetries of the system. In turn, in some cases, this extra integral may be used to prove periodicity of the reduced dynamics and hence—via established 'reconstruction' techniques from the theories of symmetric dynamical systems and Lie group—the quasi-periodicity (without the need of a time-reparametrisation) of the dynamics of the unreduced system. As an application, this mechanism is used to provide a detailed analysis of the system formed by a heavy sphere that rolls without sliding inside a uniformly rotating convex surface of revolution. (This talk is based on joint works with M. Dalla Via, L. Garcia-Naranjo and N. Sansonetto).

Thursday, 12:00

Multisymplectic approach to Symmetries in General Relativity and other field theories

Jordi Gaset
Universitat Politècnica de Catalunya, Spain

Symmetries and conserved quantities play a key role in modern physics. Their geometric interpretation give rise to different concepts: Conserved quantities (and conservation laws), gauge symmetries, rigid symmetries, Cartan (Noether) symmetries, Noether-type theorems and more. We will present them and discuss their geometrical meaning, for classical Lagrangian field theories of first and second order, using the multisymplectic formulation of these theories. As example we will use the gravitational theory; in particular, the Hilbert-Einstein formulation (second order) and the Palatini approach (first-order).

Discrete Field Theory on Cellular Complexes

Thursday, 12:30

Pablo M. Chacón
Universidad de Salamanca, Spain

In the 90s of last century J. Moser, A.P Veselov, J.E. Marsden, G.W. Patrick, S. Shkoller, among other authors, proposed to discretize the Calculus of Variations in order to approximate numerically since its same root the Euler-Lagrange equations. The result was, essentially, a discretization of the Hamilton-Cartan formalism of the calculus of variations through the so-called variational integrators, which consist in integrating the discrete Euler-Lagrange equations by means of certain families of symplectic diffeomorphisms. Three basic references on this topic, which include a well selected bibliography, are:

- J.E. Marsden and M. West, Discrete mechanics and variational integrators, Acta Numer. 10 (2001), 357–514.
- J.E. Marsden, G.W. Patrick and S. Shkoller, Multisymplectic geometry, variational integrators, and nonlinear PDEs, Comm. Math. Phys. 199 (1998), 351–395.
- A.C. Casimiro and C. Rodrigo, First variation formula and conservation laws in several independent discrete variables, J. Geom. Phys. 62 (2012), 61–86.

In these references, the base manifold of the corresponding fibration is the standard cellular complex in $\mathbb{R}^n$, for $n=1, n=2$, and arbitrary n respectively, being the basic object of the two first ones a discrete counterpart of the Cartan form in the continuous case, while in the third one a variation formula of the discrete Lagrangian density expressed exclusively in terms of the geometry of the cellular complex of the base manifold is emphasized.
In this general framework, our research group at the University of Salamanca has extended to the field theory the symplectic approach of the problem of Lagrange in discrete mechanics in the sense of:

- P.L García, A. Fernández, and C. Rodrigo, Variational integrators for discrete Lagrange problems, J. Geom. Mech. 2 (2010), 343–374.

Such an extension is based on a previous multisymplectic formulation of the unconstrained discrete field theory on cellular complexes, that will be the topic of this talk. This is a joint work with A. Fernández (USAL), P.L García (RAC and USAL), and C. Rodrigo (Acad. Militar Portugal).

Applications of Lie systems theory in classical and quantum physics

Thursday, 15:00

José F. Cariñena
Departamento de Física Teórica,
Universidad de Zaragoza, 50009
Zaragoza, Spain

After a quick presentation of the theory of Lie systems from a geometric perspective, recent progresses on their applications when compatible geometric structures exist will be described, with an special emphasis in the particular case of admissible Kähler structures, and therefore with applications in Quantum Mechanics. More specifically it will be shown that they are useful in the study of the time evolution of a quantum system, as well as in particular cases of time-independent Schrödinger equation. Applications in control theory will also be exhibited.

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2. J.F. Cariñena, J. Clemente-Gallardo and A. Ramos, *Motion on Lie groups and its applications in Control Theory*, Rep. Math. Phys. **51**, 159–170 (2003)
3. J.F. Cariñena, D.J. Fernández and A. Ramos, *Group theoretical approach to the intertwined Hamiltonians*, Ann. Phys. (N.Y.) **292**, 42–66 (2001)
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9. J.F. Cariñena, J. Grabowski and A. Ramos, *Reduction of t-dependent systems admitting a superposition principle*, Acta Appl. Math. **66**, 67–87 (2001)
10. J.F. Cariñena, A. Ibort, G. Marmo and G. Morandi, *Geometry from Dynamics, Classical and Quantum*, Springer, 2015
11. J.F. Cariñena and J. de Lucas, *Lie systems: theory, generalisations, and applications*, Dissertationes Mathematicae **479**, 2011
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14. J.F. Cariñena and A. Ramos, *A new geometric approach to Lie systems and physical applications,* Acta Appl. Math. **70**, 43–69 (2002)
15. J.F. Cariñena and A. Ramos, *Applications of Lie systems in Quantum Mechanics and Control Theory*, in: *Classical and quantum integrability*, Banach Center Publ. **59**, 143–162, Warsaw, 2003
16. J.F. Cariñena and A. Ramos, *Generalized Bäcklund-Darboux transformations in one-dimensional quantum mechanics*, Int. J. Geom. Methods Mod. Phys. **5**, 605–640 (2008)

Thursday
16:00

Stable orbits in unstable systems: the case of the Pais-Uhlenbeck oscillator

José Antonio Vallejo

Universidad Autónoma de San Luis Potosí, Mexico

In classical field theories containing derivatives of higher order, Ostrogadsky's theorem guarantees the presence in the dynamical equations of terms leading to operators with unbounded spectrum after the application of the usual quantization procedures. The corresponding unnormalized states are called ghosts. When interactions are turned on, these terms induce instabilities which ultimately collapse the system.

In spite of this, theories with higher order derivatives are very interesting from the point of view of Physics because they include renormalizable quantum gravity models, a feature that has driven an enormous amount of literature trying to make sense of ghosts. In some of these attempts it has been observed the existence, at a classical level, of 'stability islands': ranges of system's values for which the motion remain bounded and stable. In this talk I will explain this phenomenon in the case of the Pais-Uhlenbeck oscillator by applying some techniques coming from singular reduction and the theory of normal forms for perturbed Hamiltonian systems.

This is joint work with Misael Avendao-Camacho and Yury Vorobiev (ref: J. of Math. Phys. 58 (2017) 093501)

Geometric theory of flexible and expandable tubes conveying fluid

Friday, 9:30

Vakhtang Poutkaradze
University of Alberta, Canada

We present a theory for the three-dimensional evolution of tubes with expandable walls conveying fluid. Our theory can accommodate arbitrary deformations of the tube, arbitrary elasticity of the walls, and both compressible and incompressible flows inside the tube. We also present the theory of propagation of shock waves in such tubes and derive the conservation laws and Rankine-Hugoniot conditions in arbitrary spatial configuration of the tubes, and compute several examples of particular solutions. The theory is derived from a variational treatment of Cosserat rod theory extended to incorporate expandable walls and moving flow inside the tube. The geometric approach to the problem also allows writing the Poisson bracket for the system. The results presented here are useful for biological flows and industrial applications involving high speed motion of gas in flexible tubes.

The work was partially supported by NSERC and University of Alberta.

A variational structure for integrable hierarchies

Friday 10:30

Mats Vermeeren
Technische Universität Berlin, Germany

Systems in classical mechanics are usually considered integrable if they satisfy the Liouville-Arnold property of having sufficiently many Poisson-commuting Hamiltonians. The Hamiltonian point of view is also one of the most popular ways to study hierarchies of integrable PDEs. Somewhat surprisingly, there is no well-established variational theory of integrability: the Lagrangian perspective is mostly missing from the field.

A Lagrangian theory of integrability, applicable to PDEs, ODEs, and fully discrete systems, has been developed in recent years by Frank Nijhoff, Yuri Suris, and their respective coauthors. I will give an introduction to this so-called "pluri-Lagrangian" (also "Lagrangian multiform") theory of integrability, and present some of its the latest developments, which include continuum limits of lattice systems and connections to the Hamiltonian formulation of integrability and to the classical notion of variational symmetries.

Lie-Poisson reduction and reconstruction phases in the N-vortex problem

Friday 11:30

Antonio Hernández
Instituto Tecnológico Autónomo de México (ITAM), Mexico

The geometry of the dynamics of the three-point-vortex problem is strikingly similar to that of the free rigid body. As we shall discuss in this talk, both problems share some aspects of their reduction (as Lie-Poisson systems, albeit with different symmetry groups) and reconstruction phases, where we have found an expression reminiscent of Montgomery's formula for the rotation angle of a rigid body after one cycle in reduced space is completed. Extensions of these ideas for the N point-vortex problem will also be mentioned. This is a collaboration with Banavara N. Shashikanth.

Friday 12:30

Non-Abelian momentum polytopes for $SU(3)$ action on products of CP^2 with an application to generalised point vortices

Amna Shaddad
University of Manchester, UK

The momentum polytope provides geometric information on the image of the momentum map. We study a system of vortices on the complex projective plane CP^2 through the momentum polytopes of the $SU(3)$ action on them. We will demonstrate the full classification of the momentum polytopes, and respective transition polytopes, of the $SU(3)$ action on $CP^2 \times CP^2$ and $CP^2 \times CP^2 \times CP^2$. This has been achieved by describing a particular action with weighted symplectic form. For two copies the momentum polytopes are line segments. For three copies there are 8 different momentum polytopes, with 3 transition polytopes. We use these momentum polytopes to investigate the relative equilibria and reduced dynamics of systems of 2 and 3 point vortices on CP^2 by ascribing the distinct weights of the symplectic form to distinct vortex strengths. For 2 vortices, the reduced spaces are points. For 3 vortices, the reduced spaces are usually 2-spheres.

Friday, 15:00

Linearization of nonlinear connections on vector bundles

Eduardo Martínez
Universidad de Zaragoza, Spain

From a nonlinear connection on a vector bundle we will construct a linear connection. The geometric properties of the linear connection will be studied and several applications will be provided.

Friday, 16:00

Studying N-body problems with the geometry of the 'Jacobi-Maupertuis' metric

Connor Jackman
MSRI, USA

The Jacobi-Maupertuis variational principle reparametrizes solutions of a natural mechanical system on a fixed energy level as geodesics of a certain metric (JM-metric for short). This poster presents some properties of this JM-metric associated to various N-body problems.

In particular, the JM-principle seems especially suited to studying the strong force problem: when the force is inversely proportional to the cubes of the mutual distances. In this case, the JM-metric is complete and, at the zero energy level, admits an additional scaling symmetry. For three equal masses under a strong force, R. Montgomery has shown the dynamics is like a geodesic flow on a negatively curved pair of pants. For more than three bodies, one can obtain similar results by restricting to 'subproblems'. For example the collinear configurations.

Moving away from the strong force, the JM-metric takes a nice form near central configurations. Ong Chong Pin examined the sectional curvature values around the Lagrange equilateral triangle configuration, while Barbosu and Elmabsout noticed certain conditions for these curvature values to be negative, and related to instability properties. When $N > 3$, one can obtain similar negative curvature values over certain homographic motions and relate them to instability properties.

Posters

Contact Hamiltonian Dynamics: a physicist's perspective

Poster. Thursday 16:30

Alessandro Bravetti
CIMAT, Mexico

We introduce contact Hamiltonian systems, then we describe their main geometric and dynamical properties, and finally we present some applications of relevance in physics.

The dynamics of an articulated n-trailer vehicle

Poster. Thursday 16:30

Alejandro Bravo Doddoli
UC Santa Cruz, USA

We consider the dynamics of an articulated n-trailer vehicle that moves under its own inertia. Such system consists of a leading car, or truck, that is pulling n trailers, like a luggage carrier in the airport. The leading car and the trailers form a convoy that is subjected to $(n+1)$-nonholonomic constraints, one for each body. This system is a canonical example in nonholonomic motion planning, which is fundamental in robotics, and has been extensively considered from the control perspective.

We derive the reduced equations of motion for an articulated n-trailer vehicle that moves under its own inertia on the plane. We show that the energy level surfaces in the reduced space are $(n+1)$-tori and we classify the equilibria within them, determining their stability. A thorough description of the dynamics is given in the case $n = 1$. The nonholonomic constraints in the n-trailer system arise by assuming that each of the bodies in the convoy has a pair of wheels that prohibit motion in the direction perpendicular to them. Each of these constraints is identical to the nonholonomic constraint for the well-known Chaplygin sleigh problem.

1. Bravo-Doddoli A. and García-Naranjo L. C., The dynamics of an articulated n-trailer vehicle. *Reg. and Chaot. Dyn.* **20** (2015), 497–517.

Poster.
Thursday
16:30

Application of the theory of currents to dynamical systems

Inti Cruz
Universidad Nacional Autónoma de México (UNAM), Mexico

One of the classical applications of the theory of currents is to the study of the problem minimal surfaces. In our work we describe two applications of the theory in the field of dynamical systems. The first such application is to prove that the *plugs* are not geodesibles. The other determines certain properties of the homology groups of a manifold which supports an *Axiom A diffeomorphism*.

Poster.
Thursday
16:30

Generalized variational calculus for mechanical systems

Viviana Alejandra Díaz
Universidad Nacional del Sur, Bahía Blanca, Argentina

In this joint work with David Martín de Diego we have considered a formalism that generalizes variational calculus and allows us to deal with different mechanical systems within the same setting. In particular, we have described how the cases of Lagrangian and Hamiltonian mechanics, subjected to constraints or not, fit in this framework. The proposed formalism is based on the notions of the tangent lift of curves and the complete lift of a vector field. Hence it can be extended to the Lie algebroids theory, and thus it includes also the case of reduced mechanical systems.

Poster.
Thursday
16:30

Quantum dechoherence as classical unstable feedback?

Manuel de la Cruz
UAM-Iztapalapa, Mexico

If quantum states are pure, we think as being isolated by the sorroundings. In more realistic situation we do not have total knowledge about the physical system and is necesarry to use mixed states. On the other hand, is possible to implement control theory of closed quantum two-level systems using the dynamics of classical rigid body and his mathematical framework. Here we present some attemps to implement control theory using rigid bodies, cuasi-rigid bodies and deformable bodies in order to derive control fields and his relevant features for particular open quantum systems with relaxation and decoherence.

Periodic orbits in non-symmetrical symplectic maps

Poster. Thursday 16:30

David Martínez del Río
IIMAS-UNAM, Mexico

The search of high order periodic orbits in 2D symplectic maps has been done efficiently only on maps with symmetries that reduce the search from the actual 2D phase space to 1D. A modified version of the parameterization method introduced by de la Llave et al. is implemented to estimate periodic orbits in any given two dimensional symplectic map. The benchmark used to test the proposed method is a three parameter analytic ratchet-like map studied before by Simo-Olvera-Petrov that lacks of symmetry lines for a generic election of the parameters. The results show a critical manifold in the parameter space with folds and singular points.

Functoriality of Principal Bundles

Poster. Thursday 16:30

Gustavo A. Saldaña Moncada
Universidad Nacional Autónoma de México (UNAM), Mexico

Given a G-principal bundle with a principal connection, to every linear representation of G, we can associate a vector bundle with a linear connection. Above mentioned is important in physics, but from a mathematical point of view, this association between linear representations and vector bundles with linear connections is functorial. The purpose of this poster is show properties of this functor, particularly, we will give a characterization of the image on objects and on morphisms under some topological conditions.

Lagrangian reduction of nonholonomic discrete mechanical systems by stages

Poster. Thursday 16:30

Cora Tori and Marcela Zucalli
Universidad de La Plata, Argentina
(joint with J. Fernández, Bariloche)

It is well known that the reduction of mechanical systems with symmetry is a process that can be repeated. The Lagrange-Poincaré category is stable under the procedure of reduction of holonomic lagrangian systems. Also the Lagrange-D'Alembert-Poincaré category is closed under the procedure of reduction of nonholonomic mechanical systems. In the framework of discrete mechanics, in order to make the process of reduction of discrete mechanical systems stable the discrete Lagrange-Poincaré category was defined. In this work we introduce a category of discrete Lagrange-D'Alembert-Poincaré systems and study some of its properties. In particular, under some adequate hypothesis, we can define a process of reduction that is closed in this category. Furthermore, we show that the reduction in two steps (first by a closed normal subgroup of the symmetry group and then by the residual symmetry group) is isomorphic, in this category, to the reduction by the full symmetry group.

Poster.
Thursday
16:30

Unimodularity Criteria for Coupling Poisson and Dirac Structures and Their Applications

Eduardo Velasco-Barreras
Universidad de Sonora, Mexico

The modular class of a Poisson manifold is an element of the first Poisson cohomology which measures the obstruction to the existence of a volume form invariant under all Hamiltonian flows. If this class vanishes, then the Poisson manifold is said to be unimodular. As shown by Abouqateb & Boucetta, for a regular Poisson manifold, the modular class is related with the Reeb class of its symplectic foliation. We are interested in the behavior of the modular classes for the family of the so-called coupling Poisson structures on a foliated manifold, which may admit singularities. We give a geometric description of modular vector fields and formulate some unimodularity criteria for coupling Poisson structures by using the notion of a generalized Reeb class. We show also how these results can be extended to the Dirac setting. As an application, we compute the modular vector fields for the gauge Poisson structures on a Lie-Poisson bundle which appear in the context of the Hamiltonian description of Wong's type equations for a colored particle in a Yang-Mills field.

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